**Assignment 01**

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| **Exercise Lists** | Please, check the exercise lists and write down the answer of each problem (both computer typing and handwriting are possible).   1. p.13, #2 2. p.14, #16 3. p.15, #18 4. p.16, #34 5. p.17, #48 6. p.38, #12 7. p.38, #22 8. p.56, #2 9. p.56, #8 10. p.57, #12 11. p.68, #4 12. p.71, #28 |
| **Answer** | |
| 1. p.13, #2:   a) Do not pass go.  This is not a proposition, also hasn't truth values.  b) What time is it?  This is not a proposition, also hasn't truth values.  c) There are no black flies in Maine  This is a proposition, and have a truth values.  d) 4 + x = 5  This is not a proposition, also hasn't truth values.  e) The moon is made of green cheese.  This is a proposition, and have a truth values.  f) ＋≥ 100  This is not a proposition, also hasn't truth values.   1. p.14, #16:   Let p, q, and r be the propositions  p: You get an A on the final exam.  q: You do every exercise in this book.  r: You get an A in this class.  Write these propositions using p, q, and r and logical connectives (including negations).  a) You get an A in this class, but you do not do everyexercise in this book.  : ￢q∧r  b) You get an A on the final, you do every exercise inthis book, and you get an A in this class.  : p∧q∧r  c) To get an A in this class, it is necessary for you to getan A on the final.  : p→r  d) You get an A on the final, but you don’t do every exercise in this book; nevertheless, you get an A in thisclass.  : (p∨￢q)∧r  e) Getting an A on the final and doing every exercise inthis book is sufficient for getting an A in this class.  : (p∧q)∨r  f ) You will get an A in this class if and only if you eitherdo every exercise in this book or you get an A on the final.  : q↔r  3) p.15, #18  Determine whether these biconditionals are true or false.  a) 2 + 2 = 4 if and only if 1 + 1 = 2.  : T  b) 1 + 1 = 2 if and only if 2 + 3 = 4.  : F  c) 1 + 1 = 3 if and only if monkeys can fly.  : T  d) 0 > 1 if and only if 2 > 1.  : F  4) p.16, #34  Construct a truth table for each of these compound propositions.  a) p ∧ ¬p   |  |  |  | | --- | --- | --- | | p | ¬p | p ∧ ¬p | | T | F | F | | F | T | F |   b) p ∨ ¬p   |  |  |  | | --- | --- | --- | | p | ¬p | p ∨ ¬p | | T | F | T | | F | T | T |   c) (p ∨ ¬q) → q   |  |  |  |  |  | | --- | --- | --- | --- | --- | | p | q | ¬q | p ∨ ¬q | (p ∨ ¬q) → q | | T | T | F | T | T | | T | F | T | T | F | | F | T | F | F | T | | F | F | T | T | F |   d) (p ∨ q) → (p ∧ q)   |  |  |  |  |  | | --- | --- | --- | --- | --- | | p | q | p ∨ q | p ∧ q | (p ∨ q) → (p ∧ q) | | T | T | T | T | T | | T | F | T | F | F | | F | T | T | F | T | | F | F | F | F | T |   e) (p → q) ↔ (¬q → ¬p)   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | p | q | ¬p | ¬q | p → q | ¬q → ¬p | (p → q) ↔ (¬q → ¬p) | | T | T | F | F | T | T | T | | T | F | F | T | F | F | T | | F | T | T | F | T | T | T | | F | F | T | T | T | T | T |   f ) (p → q) → (q → p)   |  |  |  |  |  | | --- | --- | --- | --- | --- | | p | q | p → q | q → p | (p → q) → (q → p) | | T | T | T | T | T | | T | F | F | T | T | | F | T | T | F | F | | F | F | T | T | T |   5) p.17, #48  Evaluate each of these expressions.  a) 1 1000 ∧ (0 1011 ∨ 1 1011)  = 1 1000 ∧ 1 1011 =1 1000  b) (0 1111 ∧ 1 0101) ∨ 0 1000  =0 0101 ∨ 0 1000 =0 1101  c) (0 1010 ⊕ 1 1011) ⊕ 0 1000  =1 0001 ⊕ 0 1000 =1 1001  d) (1 1011 ∨ 0 1010) ∧ (1 0001 ∨ 1 1011)  =1 1011 ∧ 1 1011 =1 1011  6) p.38, #12  Show that each of these conditional statements is a tautology by using truth tables.  a) [¬p ∧ (p ∨ q)] → q   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | p | q | ¬p | p ∨ q | ¬p ∧ (p ∨ q) | [¬p ∧ (p ∨ q)] → q | | T | T | F | T | F | T | | T | F | F | T | F | T | | F | T | T | T | T | T | | F | F | T | F | F | T |   b) [(p → q) ∧ (q → r)] → (p → r)   |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | p | q | r | p → q | q → r | (p → q) ∧ (q → r) | (p → r) | [(p → q) ∧ (q → r)]  → (p → r) | | T | T | T | T | T | T | T | T | | T | T | F | T | F | F | F | T | | T | F | T | F | T | F | T | T | | F | T | T | T | T | T | T | T | | T | F | F | F | T | F | F | T | | F | T | F | T | F | F | T | T | | F | F | T | T | T | T | T | T | | F | F | F | T | T | T | T | T |   . . .  c) [p ∧ (p → q)] → q   |  |  |  |  |  | | --- | --- | --- | --- | --- | | p | q | p → q | p ∧ (p → q) | [p ∧ (p → q)] → q | | T | T | T | T | T | | T | F | F | F | T | | F | T | T | F | T | | F | F | T | F | T |   d) [(p ∨ q) ∧ (p → r) ∧ (q → r)] → r   |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | p | q | r | p ∨ q | p → r | q → r | (p ∨ q) ∧ (p → r) ∧ (q → r) | [(p ∨ q) ∧ (p → r)  ∧ (q → r)] → r | | T | T | T | T | T | T | T | T | | T | T | F | T | F | F | F | T | | T | F | T | T | T | T | T | T | | F | T | T | T | T | T | T | T | | T | F | F | T | F | T | F | T | | F | T | F | T | T | F | F | T | | F | F | T | F | T | T | F | T | | F | F | F | F | T | T | F | T |   7) p.38, #22  Show that p → q and ¬q → ¬p are logically equivalent.   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | p | q | ¬p | ¬q | p → q | ↔ | ¬q → ¬p | | T | T | F | F | T | T | T | | T | F | F | T | F | T | F | | F | T | T | F | T | T | T | | F | F | T | T | T | T | T |   8) p.56, #2  Let P(x) be the statement “The word x contains the letter a.”  What are these truth values?  a) P(orange) : T  b) P(lemon) : F  c) P(true) : F  d) P(false) : T  9) p.56, #8  Translate these statements into English, where R(x) is “x is a rabbit”  and H(x) is “x hops” and the domain consists of all animals.  a) ∀x(R(x) → H(x)) : For all animal, if it is a rabbit then it hops.  b) ∀x(R(x) ∧ H(x)) : For all animal that is a rabbit and it hops.  c) ∃x(R(x) → H(x)) : There is an animals, if it is rabbits then they hops.  d) ∃x(R(x) ∧ H(x)) : There is an animals that is rabbits and they hops.  10) p.57, #12  Let Q(x) be the statement “x + 1 > 2x.” If the domainconsists of all integers,  what are these truth values?  a) Q(0) : T  b) Q(−1) : T  c) Q(1) : F  d) ∃xQ(x) : T  e) ∀xQ(x) : F  f ) ∃x¬Q(x) : T  g) ∀x¬Q(x) : F  11) p.68, #4  Let P(x, y) be the statement “Student x has taken class y,”  where the domain for x consists of all students in your class  and for y consists of all computer science courses at your school.  Express each of these quantifications in English.    a) ∃x∃yP(x, y) :  There is a student in your class that has taken a computer science course.  b) ∃x∀yP(x, y) :  There is a student in your class that taken all computer science courses.    c) ∀x∃yP(x, y) :  All student in your class that have taken a computer science course.    d) ∃y∀xP(x, y) :  There is a computer science course has taken every student in your class.    e) ∀y∃xP(x, y) :  Every computer science courses has been taken by some student in your class.    f ) ∀x∀yP(x, y) :  All student in your class have taken all computer science courses.  12) p.71, #28  Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.  a) ∀x∃y( )  : This is True. The rule = determines a function, and hence the quantity exists for any x.  b) ∀x∃y( )  : This is Not true. Not applicable when x is -1.  c) ∃x∀y(xy = 0)  : This is True. If x is 0, than it comes true for any x.  d) ∃x∃y(x + y ≠ y + x)  : This is False. The addition of real numbers is commutative.  e) ∀x(x ≠ 0 → ∃y(xy = 1))  : This is True. Not applicable when y is 0.  f ) ∃x∀y(y ≠ 0 → xy = 1)  : This is False. When x is 0, any y is not applicable.  g) ∀x∃y(x + y = 1)  : This is True. set y = 1 - x.  h) ∃x∃y(x + 2y = 2 ∧ 2x + 4y = 5)  : This is False. Divide the second equation by 2, than x + 2y = 5/2. This is not the same as the first equation.  i) ∀x∃y(x + y = 2 ∧ 2x − y = 1)  : This is True. Multiply the second equation by 2, than 4x - 2y = 2.  and solving 4x - 2y = x + y, It is x = y. So, When x = y, the equation is holds.  j) ∀x∀y∃z(z = (x + y)∕2)  : This is True. the rule z = (x + y)/2 determines a function, and hence the quantity z exists for any x and y. | |
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